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Capital Quickening and the Rate of Interest

Hans Brems

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

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July 1988

Capital Quickening and the Rate of Interest

Hans Brems, Professor
Department of Economics

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HANS BREMS
Box 99 Commerce West
1206 S. Sixth Street
Champaign, IL 61820

CAPITAL QUICKENING AND THE RATE OF INTEREST*

By HANS BREMS

Abstract

According to the conventional wisdom, under embodied technological progress a durable producers' good will be operated until its quasi-rent has become zero. The present paper derives such wisdom as a special case of a model of pure replacement in which there is labor-capital substitution between old and new equipment: replacing too rarely means wasting labor on the operation of too much obsolescent equipment. Replacing too frequently means wasting capital on too much investment. The firm will strike a balance, and a lower rate of interest will tilt the balance in favor of more frequent replacement. Such shorter useful life is a reversal of static Austrian doctrine.

CAPITAL QUICKENING AND THE RATE OF INTEREST*

By HANS BREMS

I. INTRODUCTION

1. Time and Interest

The price of time is interest, and in Austrian capital-deepening models the interest elasticity of time absorbed was found to be negative.

The Böhm-Bawerk (1889) centenary is a fine occasion to revisit the interest elasticity of time. Let us replace stationary Austrian technology by embodied technological progress, "by a substantial margin the more important kind," as Solow (1962: 77) observed. We shall build the simplest possible pure-replacement model of the firm: there will be only two goods, a consumers' good and a durable producers' good, there will be only two kinds of labor, construction labor and operating labor, and there will be fixed labor-input coefficients.

Such fixed labor-input coefficients will rule out more physical capital stock per man ("capital deepening") but still leave room for an ever more productive physical capital stock per man. Let all technological progress be embodied [Johansen (1959), Solow (1960) and (1962), and Massell (1962)]. Then the firm can have such more productive physical capital stock only via replacement. A price mechanism will encourage such replacement. The ever-growing physical output per producers' good will be selling at a price common to all firms and reflecting the latest technology available to them. Such a common price will be declining relative to the money wage rate and thus exert a growing pressure to replace.

Will the rate of interest be part of such "capital quickening"?

2. Variables

I ≡ aggregate physical gross investment

J ≡ present net worth to a firm of an endless succession of vintages

j ≡ present net worth to a firm of a single vintage

L ≡ aggregate labor employed

m ≡ mark-up factor of consumers' goods industry

P ≡ price of consumers' goods

S ≡ aggregate physical capital stock of producers' goods

$u \equiv$ useful life of producers' goods

$X \equiv$ physical output of consumers' goods per annum per producers' good

3. Parameters

$a_1 \equiv$ labor absorbed in constructing one physical unit of producers' goods

$a_2 \equiv$ labor absorbed per annum in operating one physical unit of producers' goods

$F \equiv$ aggregate available labor force

$g \equiv$ rate of inflation

$q \equiv$ rate of technological progress

$r \equiv$ nominal rate of interest

$\rho \equiv$ real rate of interest

$w \equiv$ money wage rate

II. FIRM EQUILIBRIUM

1. An Endless Succession of Vintages

Let a firm plan an endless succession of vintages of durable producers' goods: every uth year a retired producers' good is replaced by a new one embodying latest technology.

Define present net worth of such an endless succession as present worth of all its future revenue minus present worth of all its future construction labor minus present worth of all its future operating labor.

2. Present Worth of All Future Revenue

Assume physical output X(v) of consumers' goods per annum per producers' good of vintage v to remain uniform throughout the useful life of that vintage. Such uniformity will rule out reswitching [Samuelson (1966)]. But let price P of consumers' goods reflect latest technology available to rivals and entrants, hence be inflating at the rate of inflation g minus the rate of technological progress q:

$$P(t) = e^{(g - q)(t - v)}P(v) \tag{1}$$

Define the real rate of interest as the nominal one minus the rate of inflation:

$$\rho \equiv r - g \tag{2}$$

As seen from time v revenue $P(t)X(v)$ at time t is $e^{-r(t - v)}P(t)X(v)$. Insert (1) and (2) and write present worth of all such future revenue throughout the useful life of vintage v as

$$\int_v^{v + u} e^{-(\rho + q)(t - v)}P(v)X(v)dt = \frac{1 - e^{-(\rho + q)u}}{\rho + q} P(v)X(v) \tag{3}$$

Every uth year, say at time t, a new vintage will embody new technology progressing at the rate q pe annum:

$$X(t) = e^{q(t - v)}X(v) \tag{4}$$

In (3) replace v by v + iu and find the future worth as seen from t = v + iu of all revenue throughout the useful life of the ith replacement:

$$\frac{1 - e^{-(\rho + q)u}}{\rho + q} P(v + iu)X(v + iu) \tag{5}$$

In (1) and (4) replace t by v + iu, insert into (5), multiply by e^{-iru} to see all such revenue from the present time t = v rather than from t = v + iu, use (2), and write the present worth of all revenue of the ith replacement:

$$\frac{1 - e^{-(\rho + q)u}}{\rho + q} e^{-i\rho u} P(v)X(v) \tag{6}$$

Finally write (6) successively for i = 0, 1, 2, ... Summing over vintages find the present worth of an endless succession of future revenues

$$\frac{1 - e^{-(\rho + q)u}}{\rho + q}(1 + e^{-\rho u} + e^{-2\rho u} + \ldots)P(v)X(v) \qquad (7)$$

The parenthesis is an endless geometrical progression with first term 1, common ratio $e^{-\rho u}$, and sum $1/(1 - e^{-\rho u})$. As a result the present worth of the endless succession of future revenues will be

$$\frac{1 - e^{-(\rho + q)u}}{\rho + q} \frac{P(v)X(v)}{1 - e^{-\rho u}} \qquad (8)$$

3. Present Worth of All Future Construction Labor

Let the length of the construction period of producers' goods be negligible. Let us be truly Ricardian and Wicksellian and assume their construction to require nothing else than labor. Let a_1 be the labor absorbed in constructing one physical unit of producers' goods, and let a_1 be a function of neither useful life nor vintage: simply let reliability dictate a physical life always longer than the useful life dictated by obsolescence.

Every uth year, say at time t, a new vintage will be constructed at a money wage rate inflating at the rate g per annum:

$$w(t) = e^{g(t - v)}w(v) \tag{9}$$

Future construction-labor cost of the ith replacement is $a_1 w(v + iu)$. In (9) replace t by v + iu, insert that, multiply by e^{-iru} to see such construction-labor cost from the present time t = v rather than from t = v + iu, use (2), and write its present worth

$$e^{-i\rho u}a_1 w(v) \tag{10}$$

Finally write (10) successively for i = 0, 1, 2, ... Summing over vintages, find the present worth of an endless succession of future construction labor

$$(1 + e^{-\rho u} + e^{-2\rho u} + \ldots)a_1 w(v)$$

The parenthesis is the same endless geometrical progression as that of (7). As a result the present worth of the endless succession of future construction labor is

$$\frac{a_1 w(v)}{1 - e^{-\rho u}} \tag{11}$$

4. Present Worth of All Future Operating Labor

Let a_2 be labor absorbed uniformly per annum in operating one physical unit of producers' goods. Such uniformity will rule out reswitching [Samuelson (1966)].

As seen from time v operating labor cost $a_2 w(t)$ at time t is $e^{-r(t - v)} a_2 w(t)$ or, with (2) and (9) inserted, $e^{-\rho(t - v)} a_2 w(v)$. Present worth of all such future operating labor is then

$$\int_v^\infty e^{-\rho(t - v)} a_2 w(v) dt = \frac{a_2 w(v)}{\rho} \tag{12}$$

5. Present Net Worth of Endless Succession of Vintages

Define present net worth J(v) of our endless succession of vintages as present worth (8) of its future revenue minus present worth (11) of its future construction labor minus present worth (12) of its future operating labor:

$$J(v) \equiv \left[\frac{1 - e^{-(\rho + q)u}}{(\rho + q)(1 - e^{-\rho u})}\, m - \frac{a_1}{a_2}\,\frac{1}{1 - e^{-\rho u}} - \frac{1}{\rho}\right] a_2 w(v) \qquad (13)$$

where

$$m(t) \equiv \frac{P(t)X(t)}{a_2 w(t)} \qquad (14)$$

Divide numerator and denominator of (14) by X(t) and find m to be simply the mark-up factor of the consumers' goods industry, best known from post-Keynesian models but equally applicable to neoclassical ones, Brems (1979). Upon (14) use (1), (4), and (9), and find $m(t) = m(v)$: over the vintages the mark-up factor is stationary. As we did in (13), then, we may drop its time coordinate.

"Firm equilibrium" [Burmeister-Dobell (1970: 95) requires present net worth (13) to be maximized.

6. Optimal Useful Life of an Endless Succession of Vintages

Treating the mark-up factor m as a parameter given to it by pure competition or price leadership, let the firm maximize present net worth (13) of an endless succession of vintages with respect to useful life u. The first-order transcendental condition for such such a maximum is

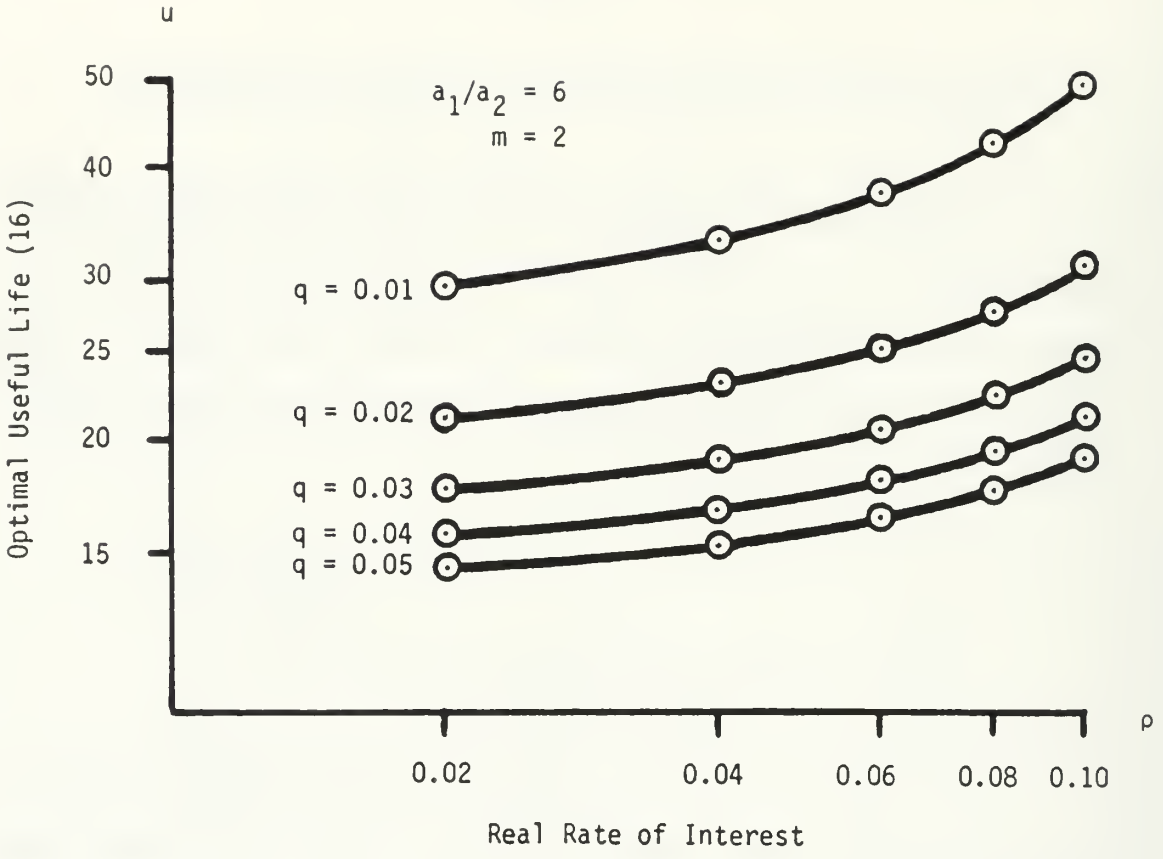
$$\frac{\partial J(v)}{\partial u} = a_2 m w(v) \frac{e^{-\rho u}}{(1 - e^{-\rho u})^2} \frac{\rho}{\rho + q} H = 0 \tag{15}$$

satisfied by

$$H \equiv (1 + \frac{1 - e^{-\rho u}}{\rho} q)e^{-qu} - 1 + \frac{a_1}{a_2} \frac{1}{m} (\rho + q) = 0 \tag{16}$$

That the second-order condition for a maximum (13) is satisfied is shown in the appendix.

Mapping the function (16) for the technological parameter $a_1/a_2 = 6$ and the marketing parameter $m = 2$ in the domains



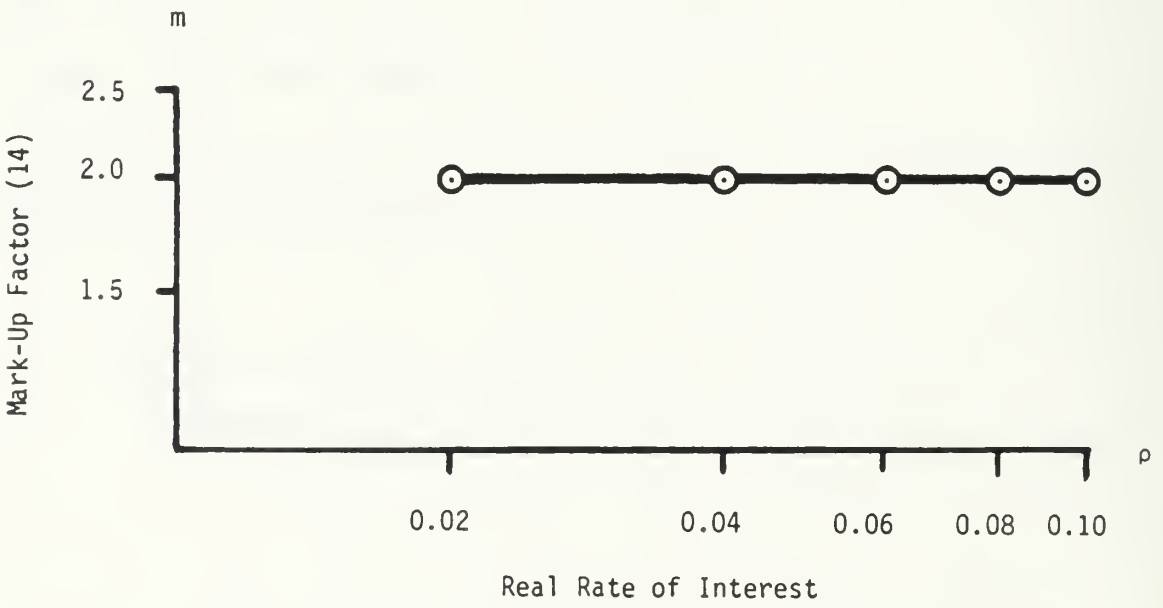


FIGURE 1. FIRM EQUILIBRIUM; ENDLESS SUCCESSION OF VINTAGES

$0.01 < q < 0.05$ and $0.02 < \rho < 0.10$, figure 1 shows optimal useful life u to be declining with a rising rate of technological progress q but rising moderately with the real rate of interest ρ--a rather un-Austrian result!

7. Present Net Worth of a Single Vintage

According to conventional wisdom [Matthews (1964: 167), Solow, Tobin, von Weizsäcker, and Yaari (1966: 79, 83, and 85), and Bardhan (1969: 315)] a durable producers' good will be operated until its quasi-rent has become zero.

Let us derive such wisdom as a special case of our pure-replacement model and begin by collapsing the latter to a single vintage, the first one, v.

As for revenue throughout the useful life of the first vintage v, we have already found its present worth (3).

As for construction labor of the first vintage v, its present worth was simply $a_1 w(v)$.

As for operating labor throughout the useful life of the first vintage v, its present worth is found by replacing "∞" in the integral (12) by $v + u$, thus writing the integral

$$\int_{v}^{v+u} e^{-\rho(t-v)} a_2 w(v)\,dt = \frac{1 - e^{-\rho u}}{\rho} a_2 w(v) \tag{17}$$

Define the present net worth j(v) of such a single vintage as the present worth of all its future revenue minus present worth of its construction labor minus present worth of all its future operating labor:

$$j(v) \equiv \left[\frac{1 - e^{-(\rho + q)u}}{\rho + q} m - \frac{a_1}{a_2} - \frac{1 - e^{-\rho u}}{\rho}\right] a_2 w(v) \tag{18}$$

where m is the mark-up factor (14). Again firm equilibrium requires such present net worth to be maximized.

8. Optimal Useful Life of a Single Vintage

Treating the mark-up factor m as a parameter given to it by pure competition or price leadership, let the firm maximize present net worth (18) of a single vintage with respect to useful life u. The first-order transcendental condition for such a maximum is

$$\frac{\partial j(v)}{\partial u} = [e^{-(\rho + q)u_m} - e^{-\rho u}]a_2 w(v) = 0 \tag{19}$$

satisfied by

$$e^{qu} = m \tag{20}$$

and easily solved for optimal useful life

$$u = \frac{\log_e m}{q} \tag{21}$$

That the second-order condition for a maximum (18) is satisfied is shown in the appendix.

Mapping the function (20) for the marketing parameter $m = 2$ in the usual domains, figure 2 shows that there is capital quickening alright, but of such quickening the rate of interest is no part: figure 2 is a system of horizontal lines.

Our (20), mapped in figure 2, is nothing else than conventional wisdom in disguise. To remove the disguise replace t by v in (14) and insert into (20):

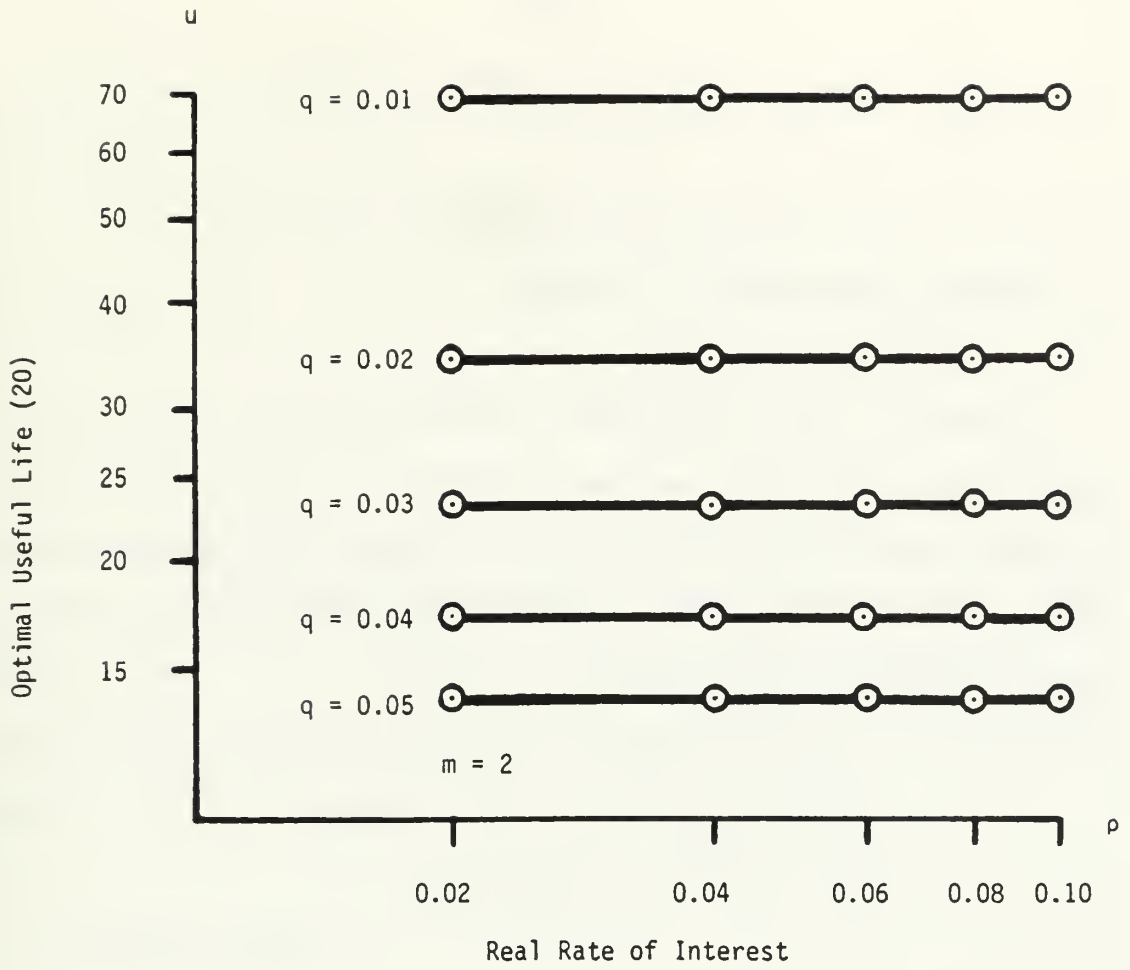
$$P(v)X(v) = e^{qu}a_2 w(v) \tag{22}$$

Then in (1) and (9) replace t by v + u, insert them into (22), divide both sides by $e^{-(g - q)u}$, and find conventional wisdom

$$P(v + u)X(v) = a_2 w(v + u) \tag{23}$$

or, in English, keep a durable producers' good of vintage v under embodied technological progress operating until its current revenue equals its current cost of operation: until its quasi-rent has become zero.

Our figures 1 and 2 are badly at odds. But we are not done yet; we must also consider equilibrium on the investment market.



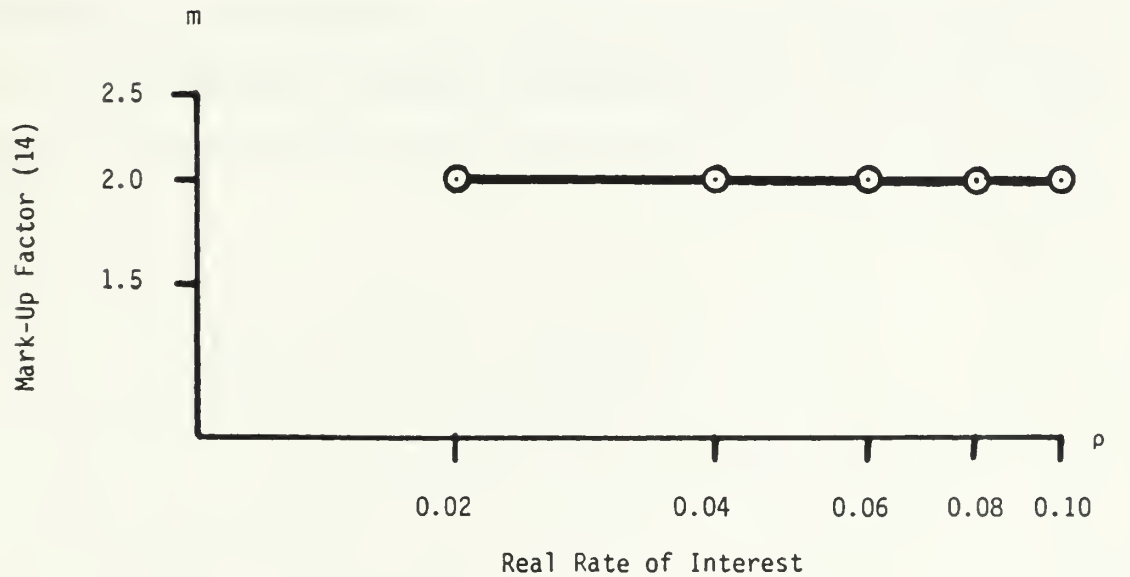


FIGURE 2. FIRM EQUILIBRIUM; SINGLE VINTAGE

III. INVESTMENT-MARKET EQUILIBRIUM

1. An Endless Succession of Vintages

"Investment-market equilibrium" [Burmeister-Dobell (1970: 95)] requires not only present net worth $J(v)$ to be maximized but also such maximized present net worth to be zero. Turning our mark-up factor m into the equilibrating variable on the investment market we set (13) equal to zero and solve for m:

$$m = \left(\frac{a_1}{a_2} + \frac{1 - e^{-\rho u}}{\rho}\right) \frac{\rho + q}{1 - e^{-(\rho + q)u}} \tag{24}$$

At simultaneous firm equilibrium and investment-market equilibrium (16) and (24) must hold simultaneously. So insert (24) into (16), rearrange, and find the transcendental equation in u alone:

$$(1 + \frac{a_2}{a_1}\frac{1 - e^{-\rho u}}{\rho})[(1 + \frac{1 - e^{-\rho u}}{\rho} q)e^{-qu} - 1]$$

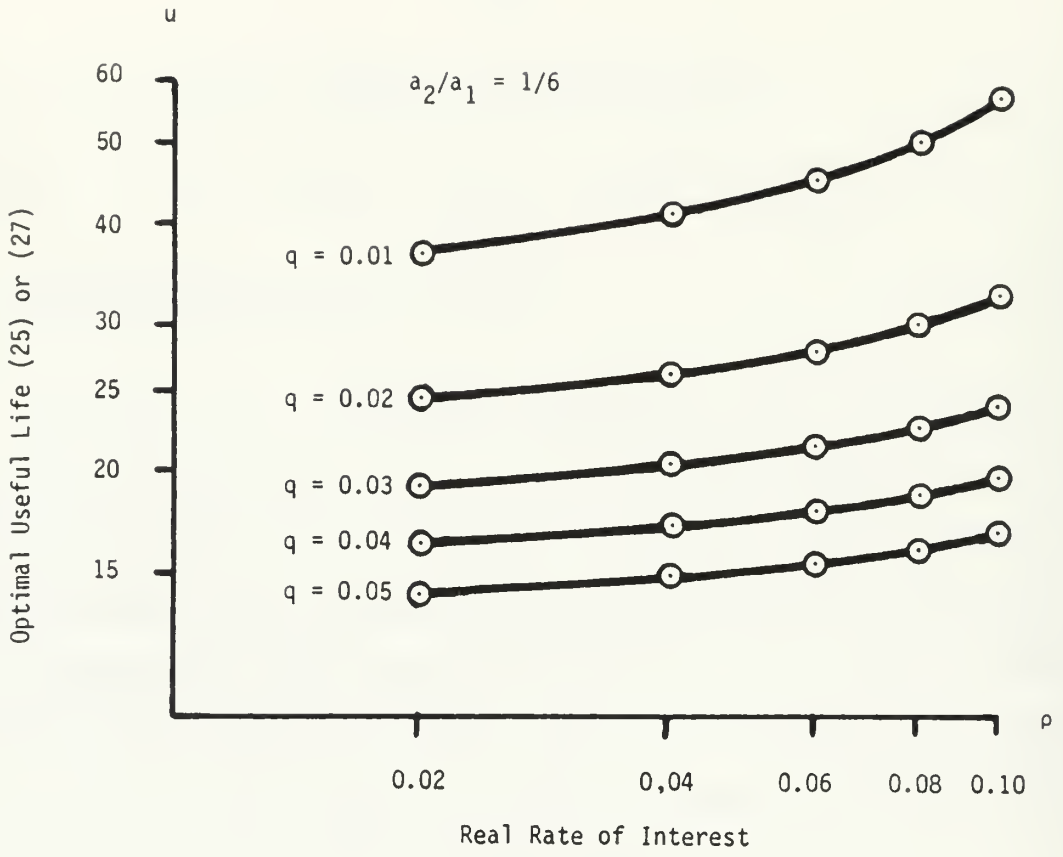
$$+ 1 - e^{-(\rho + q)u} = 0 \qquad (25)$$

Mapping the function (25) for the variable m (24) figure 3 shows optimal useful life u to be again declining with a rising rate of technological progress q and again rising moderately with the real rate of interest ρ--again a rather un-Austrian result!

2. A Single Vintage

Again investment-market equilibrium requires not only present net worth j(v) to be maximized but also such maximized present net worth to be zero. Turning our mark-up factor m into the equilibrating variable on the investment market we set (18) equal to zero and solve for m:

$$m = (\frac{a_1}{a_2} + \frac{1 - e^{-\rho u}}{\rho}) \frac{\rho + q}{1 - e^{-(\rho + q)u}} \qquad (26)$$



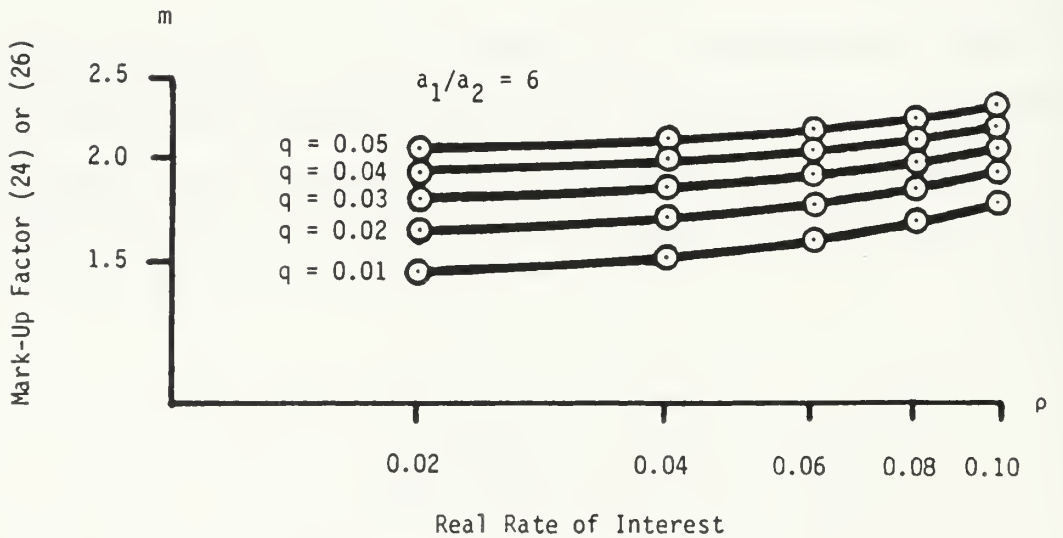


FIGURE 3. INVESTMENT-MARKET EQUILIBRIUM

EQUATION (25): ENDLESS SUCCESSION OF VINTAGES

EQUATION (27): SINGLE VINTAGE

We notice that (24) and (26) are equal and shall see why in sec. 3.

At simultaneous firm equilibrium and investment-market equilibrium (20) and (26) must hold simultaneously. So insert (26) into (20), rearrange, and find the transcendental equation in u alone:

$$\left(\frac{a_1}{a_2} + \frac{1 - e^{-\rho u}}{\rho}\right)e^{-qu}(\rho + q) - [1 - e^{-(\rho + q)u}] = 0 \qquad (27)$$

As soon as we start mapping the function (27) we find an exact replica of figure 3 mapping (25). A simple algebraic proof that (25) and (27) have identical roots is the following. Equation (25) applied to an endless succession optimized according to (16), so use (16) to express 1/m. Equation (27) applies to a single vintage optimized according to (20), so use (20) to express 1/m. Equation (24) holds in either case; consequently the two expressions of 1/m are equal. Set them equal, insert the result into (27), and arrive at (25).

A direct comparison between our endless succession and our single vintage will now be useful.

3. Endless Succession and Single Vintage Compared

Multiply both sides of the identity (13) by $(1 - e^{-\rho u})$, insert the identity (18) and see the simple relationship between our two present net worths:

$$j(v) \equiv (1 - e^{-\rho u})J(v) \tag{28}$$

We can now see why (24) and (26) were equal. Restrict ρu to $0 < \rho u < \infty$, consequently $0 < 1 - e^{-\rho u} < 1$, and $j(v)$ will be zero if and only if $J(v)$ is zero. Next differentiate (28) with respect to useful life u:

$$\frac{\partial j(v)}{\partial u} \equiv (1 - e^{-\rho u}) \frac{\partial J(v)}{\partial u} + J(v)e^{-\rho u}\rho \tag{29}$$

Under our restriction on ρu neither $1 - e^{-\rho u}$ nor $e^{-\rho u}$ in (29) can be zero, and everything we have said falls neatly into place.

In an endless succession a mere firm equilibrium $\partial J(v)/\partial u = 0$ without investment-market equilibrium: $J(v) \gtrless 0$ will make only the first term on the right-hand side of (29) equal to zero, consequently

will not imply firm equilibrium in the single vintage: $\partial j(v)/\partial u \gtrless 0$. In English: the useful life maximizing the present net worth of an endless succession will not be the same as the useful life maximizing the present net worth of a single vintage. In geometry: figures 1 and 2 were badly at odds.

By contrast, in an endless succession simultaneous firm equilibrium $\partial J(v)/\partial u = 0$ and investment-market equilibrium $J(v) = 0$ will make both terms on the right-hand side of (29) equal to zero, consequently will indeed imply firm equilibrium in the single vintage $\partial j(v)/\partial u = 0$. In English: the useful life maximizing the present net worth of an endless succession will be the same as the useful life maximizing the present net worth of a single vintage. In geometry: figure 3 will display the single-vintage approach as an exact replica of the endless-succession approach.

IV. CONCLUSIONS: TIME AND INTEREST

1. Zero Interest Elasticity of Time

Assuming firm equilibrium and a single vintage our solution (20), mapped as the horizontal lines of figure 2, showed optimal useful life

to be invariant with the rate of interest. But such invariance could survive the removal of neither assumption: our (27) generated the curvatures of figure 3 as soon as investment-market equilibrium was considered; our (16) generated the curvatures of figure 1 as soon as an endless succession of vintages was considered.

2. Negative Interest Elasticity of Time

The Austrian theory of time and interest was a theory of capital deepening, pioneered for circulating capital by Böhm-Bawerk (1889) and for fixed capital by Åkerman (1923) and Wicksell [1923 (1934)], endorsed by Blitz (1958) and Kleiman-Ophir (1966), and restated by Brems (1989). Always assuming a stationary technology, Austrians found a negative interest elasticity of time.

3. Positive Interest Elasticity of Time

Also assuming a stationary technology, the reswitching debate, summarized by Samuelson (1966), found that within already known technologies a time span might be optimal at a high rate of interest, inoptimal at a medium rate, and optimal once again at a low rate.

Breaking out of the stationary technology of the Austrians and the reswitchers, we have built a pure-replacement model showing in its figures 1 and 3 a positive but moderate interest elasticity of time.

We have kept our model very simple. Intuitively restated, its basic idea was this. On the margin of replacement there was labor-capital substitution between old and new equipment: replacing too rarely meant wasting labor on the operation of too much obsolescent equipment. Replacing too frequently meant wasting capital on too much investment. The firm would strike a balance, and a lower rate of interest would tilt the balance in favor of more frequent replacement.

We have kept our model very pure. From reswitching we have isolated it by assuming, realistically we believe, uniformity over time of both physical output and operating-labor input. From capital deepening we have isolated it by assuming fixed labor-input coefficients, as did Solow, Tobin, von Weizsäcker, and Yaari (1966: 79-80). Now at the blueprint stage capital deepening may save operating labor a_2 by more construction labor a_1. Then if labor and capital are close enough substitutes a negative Austrian interest elasticity may overwhelm our positive but moderate un-Austrian elasticity [Matthews (1964: 168, 170) and Bardhan (1969: 316)].

4. Investment and Interest

Our reversal of Austrian doctrine is fully compatible with, indeed adds new substance to, the orthodox belief that a lower rate of interest induces larger investment. Let us conclude by showing how.

Aggregate labor employed per annum in construction is

$$L_1 = a_1 I \tag{30}$$

Aggregate labor employed per annum in operation is

$$L_2 = a_2 S \tag{31}$$

Let embodied technological progress be our sole source of growth: physical capital stock will become ever more productive but remain stationary in size. Such a stationary capital stock will consist of u vintages, each of size I:

$$S = Iu \tag{32}$$

Let there be full employment:

$$F = L_1 + L_2 \tag{33}$$

Insert (30) through (32) into (33) and find physical gross investment

$$I = \frac{F}{a_1 + a_2 u} \tag{34}$$

where a_1, a_2, and F are parameters. Consequently a lower real rate of interest ρ inducing via (16), (25), or (27) a shorter optimal useful life u will induce via (34) a larger physical gross investment, as shown in figure 4.

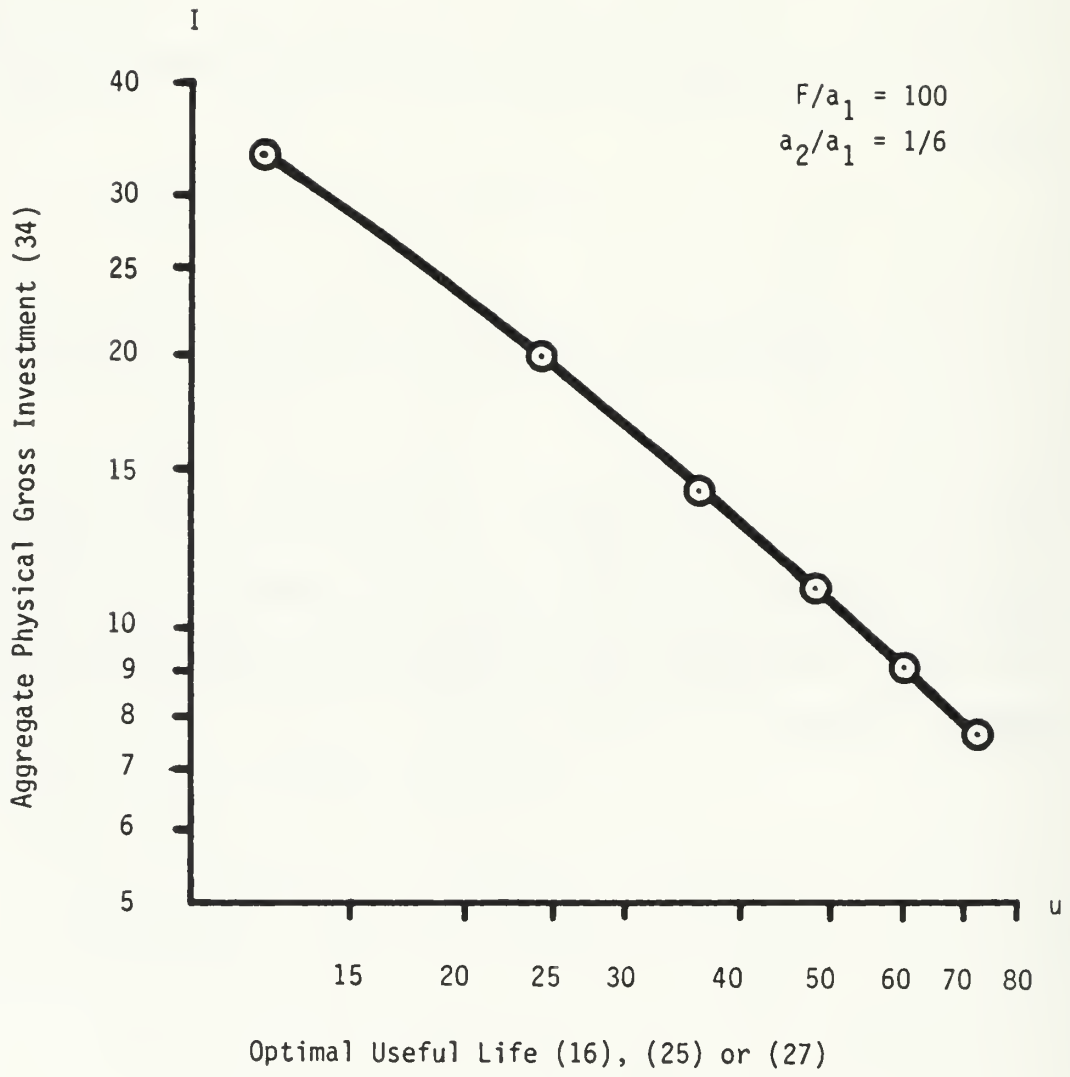


FIGURE 4. AGGREGATE PHYSICAL GROSS INVESTMENT AS A FUNCTION OF USEFUL LIFE

FOOTNOTE

*To solve our transcendental equations for small values of ρ and q a modification of the Newton-Raphson method was desirable. For such modification and for computing the solution the author is indebted to Dr. Daniel P. Connors of the Coordinated Science Laboratory of the University of Illinois. For helpful comments the author is indebted to members of workshops at the Stockholm Industrial Institute for Economic and Social Research, at the University of Gothenburg, at Duke, and at Illinois.

APPENDIX: SECOND-ORDER CONDITIONS

Differentiate (15) once more, insert first-order condition (16), and find

$$\frac{\partial^2 J(v)}{\partial u^2} = - a_2 \frac{e^{-(\rho + q)u}}{1 - e^{-\rho u}} mqw(v) < 0 \tag{35}$$

Differentiate (19) once more, insert first-order condition (20), and find

$$\frac{\partial^2 j(v)}{\partial u^2} = - a_2 e^{-(\rho + q)u} mqw(v) < 0 \tag{36}$$

Both second-order conditions are satisfied, and both (13) and (18) are indeed maximized.

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